where α, β, γ, ... are the six different prime factors 2, 3, 5, 7, 31, 61 (with repetition 2 and 5) combined in products of 1, 2, 3, ..., 10 letters at a time.

By actual calculation and checking we have found the following ten solutions:

$x=-3343$, 12721, 320, 305, 7547, -2225, -710, -3215, -8495, -56950;

$y=2$, 8, 29, 374, 4187, -1, -91, -1486, -4276, -29851;

$u=568$, -2634, -116, 454, 5923, $+337$, -126, -2136, -6102, -42446;

$v=-137$, -89, -101, 319, 5473, -141, -261, -2121, -5841, -39941.

Also solved by G. B. M. Zerr.

169. Proposed by R. D. CARMICHAEL, Princeton University.

Let $Q_n(x)=0$ be the equation whose roots are all the primitive nth roots of unity without repetition. In $Q_n(x)=0$ replace x by α/β, a fraction in its lowest terms, and clear of fractions. Let $Q_n(\alpha, \beta)$ represent the resulting first member. Set $n=mp$ where p is the largest prime factor of n. It is required to find all the integral values of α, β, m, p satisfying the following relations:

$$(1)\quad Q_{mp}(\alpha, \beta)=p,$$
$$(2)\quad \alpha^m-\beta^m\equiv 0 \pmod{p}.$$

One such solution is: $\alpha=2$, $\beta=1$, $m=2$, $p=3$. (See MONTHLY, Vol. XII, p. 89.)

[No solution of this problem has been received.]

170. Proposed by PATRICK WALSH, 1451 Annunciation Street, New Orleans, La.

The areas of rectangles A and B are respectively 15170 10/27 and 31230.3627. Find the sides and diagonal of each rectangle in exact or rational numbers.

Solution by B. F. FINKEL, Ph. D.

For A, let x and y be the dimensions of the field. Then

$$xy=15170\tfrac{10}{27}=\frac{2^{14}.5^2}{3^3}\ldots(1),\text{ and } \sqrt{(x^2+y^2)}=d\ldots(2),$$

where d is the diagonal, which is to be rational. Solving (1) for y and substituting the value thus found in (2) and reducing, we have

$$\frac{\sqrt{(3^6x^4+2^{28}.5^4)}}{3^3y}=d.$$

Let $x=\dfrac{2^7.5}{3^2}z$. Then $d=\dfrac{2^7.5}{3^2z}\sqrt{(z^4+9)}$. Let $\sqrt{(z^4+9)}=z^2t-3$. Then $z^2=\dfrac{6t}{t^2-1}$.

We must now find such values for t as will make $\frac{6t}{t^2-1}$ a perfect square. These values are 0, 1, 2, 3, $\frac{1}{2}$, $\frac{1}{3}$. For $t=2$, $z=2$,

$$d=\frac{2^6.5^2}{3^2},\ x=\frac{2^8.5}{3^2},\ \text{and}\ y=\frac{2^6.5}{3}.$$

For $t=3$, $z=\frac{3}{2}$, $d=\frac{2^6.5^3}{3^2}$, $x=\frac{2^6.5}{3}$, and $y=\frac{2^8.5^2}{3^2}$.

The values of 0 and 1 for t, give reciprocal limiting values for d, x, and y.

A similar treatment for B leads to the value, $z=2\sqrt{\frac{15t}{t^2-1}}$.
z is rational for $t=0$, 1, 4, $-\frac{1}{4}$, giving the values $z=0$, ∞, 4.
The values of x and y corresponding to $z=4$, are

$$x=\frac{3^2.19.179}{2^3\times5},\ y=\frac{3\times19.179}{5^2},\ \text{diagonal}=\frac{3^2.17.19.179}{2^3\times5}.$$

171. Proposed by PROFESSOR E. B. ESCOTT, Ann Arbor, Mich.

Solve completely:
$$2x^2-1=y,$$
$$2y^2-1=z,$$
$$2z^2-1=w,$$
$$2w^2-1=x.$$

I. Solution by PROFESSOR L. E. DICKSON, Ph. D., The University of Chicago.

It will be shown that the only integral solution is $x=y=z=w=1$.

If $x=0$ or ±1, then $y^2=1$, $z=1$, $w=1$, so that the fourth equation gives $x=1$.

Next, let $x^2>1$. Then $y>x^2$, $z>y^2>1$, $w>z^2>1$, $x>w^2$.
Hence, $x>x^{16}$, which contradicts $x^2>1$.

Also similarly solved by G. B. M. Zerr, and S. G. Barton.

II. Solution by the PROPOSER.

Let $x=\cos\phi$; then $y=\cos2\phi$, $z=\cos4\phi$, $w=\cos8\phi$, $x=\cos16\phi$.
Since $\cos16\phi=\cos\phi$, we have, $\cos16\phi-\cos\phi=0$, which may be written

$$-2\sin\tfrac{17}{2}\phi.\sin\tfrac{15}{2}\phi=0.$$

If $\sin\frac{17}{2}\phi=0$, $\phi=\frac{2n\pi}{17}$. If $\sin\frac{15}{2}\phi=0$, $\phi=\frac{2n\pi}{15}$.